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## AN ELEMENTARY DEDUCTION OF TAYLOR'S FORMULA.

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In looking over some of my mathematical notes written ten years ago, I came across the following attempt at a deduction of Taylor's formula which was then considered to lack rigor. Upon re-reading it, its novelty seems to possess sufficient interest to warrant its publication.

Let  $f x$  represent a function of the real variable  $x$ , which is uniform, finite, and continuous as are also its successive derivatives throughout an interval between  $x = \alpha$  and  $x = \beta$ .

Let  $a$  be any arbitrarily chosen point in the interval  $(a\beta)$ , and  $x$  any variable point in this interval ( $a < x$ ).

By Lagrange's form of Rolle's theorem, we have

$$fx - fa = (x - a)f'u_1, \quad a < u_1 < x$$

But since the derivatives of  $f x$  are also uniform, finite and continuous throughout  $(\alpha \beta)$ , we must have, by the same theorem

$$f'u_1 - f'a = (u_1 - a)f''u_2, \quad a < u_2 < u_1$$

$$f''u_2 - f''a = (u_2 - a)f'''u_3, \quad a < u_3 < u_2$$

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$$f^n u_n - f^n a = (u_n - a) f^{n+1} u_{n+1}. \quad a < u_{n+1} < u_n$$

Multiply this set of equalities respectively by

$$(x-a)(u_0-a)(u_1-a)\dots(u_r-a). \quad (u_0-a \equiv 1)(r=0\dots n-1)$$

Then, by addition, we have

$$\begin{aligned}fx &= fa + (x - a)f'a + (x - a)(u_1 - a)f''a + \dots \\ &\quad + (x - a)(u_1 - a) \dots (u_{n-1} - a)f^na \\ &\quad + (x - a)(u_1 - a) \dots (u_n - a)f^{n+1}u_{n+1}.\end{aligned}$$

$u_r$  is some function of  $x$ , and is equal to  $a$  when  $x = a$ . Whence  $u_r - a$  is a function of  $x$  which vanishes when  $x = a$ . We may therefore write

$$u_j - a = (x - a) \psi_j x,$$

wherein  $\phi_r x$  is some function of  $x$ . We then have

$$\begin{aligned}(x-a)(u_1-a)\dots(u_r-a) &= (x-a)^{r+1} \phi_1 x \phi_2 x \dots \phi_r x \\ &= (x-a)^{r+1} \varphi_{r+1} x,\end{aligned}$$

wherein

$$\varphi_{r+1} x = \phi_1 x \phi_2 x \dots \phi_r x$$

is to be determined.

Substituting, we have

$$\begin{aligned}f x &= f a + (x-a) f' a + (x-a)^2 \varphi_2 x f'' a + \dots \\ &\quad + (x-a)^n \varphi_n x f^n a + (x-a)^{n+1} \varphi_{n+1} x f^{n+1} u_{n+1}.\end{aligned}$$

Differentiate this equality successively with respect to  $x$ , and in the results put  $x = a$ . We obtain

$$\varphi_2 a = \frac{1}{2!}; \quad \varphi_3 a = \frac{1}{3!}; \quad \dots; \quad \varphi_{n+1} a = \frac{1}{(n+1)!}.$$

But  $a$  is any arbitrary point in the interval  $(a\beta)$ , therefore these are the values of the  $\varphi$  functions throughout the interval  $(a\beta)$ . Hence

$$f x = f a + (x-a) f' a + \dots + \frac{(x-a)^n}{n!} f^n a + \frac{(x-a)^{n+1}}{(n+1)!} f^{n+1} u_{n+1}.$$

UNIVERSITY OF VIRGINIA, Nov., 1893.